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On D-branes in the Nappi-Witten and GMM gauged WZW models

Gor Sarkissian

The Abdus Salam International Centre for Theoretical Physics
Strada Costiera 11, Trieste 34014, Italy
E-mail: gor@ictp.trieste.it

Abstract: We construct D-branes in the Nappi-Witten (NW) and Guadagnini-Martellini-Mintchev (GMM) gauged WZW models. For the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)/\mathrm{U}(1) \times \mathrm{U}(1)$ NW and $\mathrm{SU}(2) \times \mathrm{SU}(2)/\mathrm{U}(1)$ GMM models we present thte explicit equations describing the D-brane hypersurfaces in their target spaces. In the latter case we show that the D-branes are classified according to the Cardy theorem. We also present the semiclassical mass computation and find its agreement with the CFT predictions.

Keywords: D-branes, Conformal Field Models in String Theory.

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1. Introduction

In recent years, some progress in the understanding of branes on the target spaces of gauged WZW models has been made. Using the lagrangian approach to WZW models developed in [1, 2, 3], in [4, 5] D-branes in the vectorially gauged WZW models were constructed.

But it was shown in [6] that in order to construct gauge invariant WZW model it is enough to satisfy the condition

$$\mathrm{Tr}(T_{a,L}T_{b,L} - T_{a,R}T_{b,R}) = 0\,, \tag{1.1}$$

where $T_{a,L}$ and $T_{a,R}$ are any generators of the left and right embedding of the gauge group. The vectorial and axial gauging correspond to the trivial solutions of (1.1) $T_{a,L} = T_{a,R}$ and $T_{a,L} = -T_{a,R}$. Consequently it is an intersting problem to find boundary conditions invariant under gauge transformations providing the general solution of (1.1).

In [7, 8] new boundary conditions were found admitting axial gauging for an abelian gauge group. In [9], the DBI action of the D-branes defined by these boundary conditions was computed for the group SU(2).

Here, using the D-branes found in those works, and their diagonal embedding in products of groups, suggested in [10], we present boundary conditions invariant under an asymmetric action of an abelian gauge group used in the Nappi-Witten [11] and Guadagnini-Martellini-Mintchev [12, 13] models. Recently these models have attracted a lof of attention. The Nappi-Witten model was used to construct cosmological model of the pre big-bang class [14]. The Guadagnini-Martellini-Mintchev model provides an example of the $T^{p,q}$ spaces [15]. We believe that the study of the D-brane dynamics in these models will shed additional light on their properties.

This paper is organised in theq following way.

In section 2 we review some facts about non-maximally-symmetric D-branes necessary for further use.

In section 3 we present D-branes in the Nappi-Witten model, construct the action with these boundary conditions and check gauge invariance.

In section 4 we study in detail D-branes in the Nappi-Witten model considered in [14] and present the explicit equations of the corresponding hypersurfaces.

In section 5, in a similar way D-branes in the Guadagnini-Martellini-Mintchev model are considered .

In section 6 we consider in detail D-branes in the SU(2) × SU(2)/ U(1) GMM model. We show that D-branes are classified according to the Cardy theorem. We also present a semiclassical mass computation and check its agreement with the CFT prediction.

2. D-branes on group

In this section we briefly review for further use the results of [7, 9] on the non-maximally symmetric D-branes on group manifolds.

It was shown in [1] that in order to have a well-defined lagrangian action of the WZW theory on a world-sheet with boundary, the boundary conditions should satisfy the following two requirements:

1. 1. The restriction of the WZW three-form to the D-branes defined by the boundary conditions should belong to a trivial cohomology class, i.e. there should exist a two-form $\omega^{(2)}$ satisfying the equation

$$\omega^{\mathrm{WZ}}(g)_{|\mathrm{brane}} = d\omega^{(2)}\,. \tag{2.1}$$

   It was shown that given a two-form satisfying (2.1) the action can be written in the form

$$S = S(g,k) - \int_D \omega^{(2)}\,, \tag{2.2}$$

   where

$$S(g,k) = \frac{k}{4\pi}\left[\int_\Sigma d^2 z L^{\mathrm{kin}} + \int_B \omega^{\mathrm{WZ}}\right] \tag{2.3}$$

   is the usual WZW action, D is an auxiliary disc joined to Σ along the boundary, completing it to the closed manifold, and B is a three-manifold satisfying the condition $\partial B = \Sigma + D$. It was understood in [16, 17] that the two-form $\omega^{(2)}$ is equal to the antisymmetric part of the matrix giving the DBI action:

$$S_{\mathrm{DBI}} = \int \sqrt{\det(G + \omega^{(2)})}\,, \tag{2.4}$$

   or $\omega^{(2)} = B + F$.

2. Some global topological restrictions may arise from the requirement of the independence of the action (2.2) from the actual position of the embedding of the auxiliary disk in the group manifold. We don't discuss here these conditions, just stating the results. The details can be found, for example, in [3].

It was found in [2], that maximally-symmetric solutions to these conditions are quantized conjugacy classes: $C = hfh^{-1}$. The mentioned two-form was found and has the form

$$\omega^f(h) = h^{-1}dhfh^{-1}dhf^{-1}\,. \tag{2.5}$$

Global topological restrictions demand that for compact groups f is quantized and equals

$$f = e^{i\frac{2\pi}{k}\Lambda\cdot H}\,, \tag{2.6}$$

where Λ are the heighest weights and H are the Cartan algebra generators. For example, in the case of the SU(2) group important for us the branes are given by the quantized set

$$f = e^{i\hat{\psi}\sigma_3}\,, \tag{2.7}$$

where

$$\hat{\psi} = \frac{2\pi j}{k}\,, \qquad j = 0\ldots\frac{k}{2}\,. \tag{2.8}$$

These D-branes correspond to the Cardy boundary states of the corresponding CFT model [18]. In [19] the non-maximally symmetric set of boundary states was found, and in [7] their description in the lagrangian formalism was suggested. It was shown that the corresponding D-branes can be defined as the product of a U(1) subgroup and a conjugacy class $g_{\text{boundary}} = mC = mhfh^{-1}$, where $m \in \mathrm{U}(1)$. The corresponding two-form was found to be

$$\omega^{(2)}(m,h) = \omega^f(l) - \mathrm{Tr}\left(m^{-1}dmdCC^{-1}\right). \tag{2.9}$$

The topological restrictions demand that conjugacy classes belong to the same quantized set (2.6). In section 6 we will need some details about the non-maximally symmetric D-branes on SU(2). In [9], the two-form (2.9) was computed for the case of branes on SU(2). In the Euler angle parametrisation

$$g = e^{i\chi\frac{\sigma_3}{2}}e^{i\tilde{\theta}\frac{\sigma_1}{2}}e^{i\varphi\frac{\sigma_3}{2}}\,, \tag{2.10}$$

it was shown that

$$\omega^2(m,h) = B + F = \frac{\cos\hat{\psi}\tan\frac{\tilde{\theta}}{2}}{\sqrt{\cos^2\frac{\tilde{\theta}}{2} - \cos^2\hat{\psi}}}d\tilde{\theta}\wedge(d\chi - d\varphi) - 2\sin^2\frac{\tilde{\theta}}{2}d\chi\wedge d\varphi. \tag{2.11}$$

3. D-branes in the Nappi-Witten model

Let us consider the gauged WZW model G/H defined in the following way [11]. One takes $G = G_1 \times G_2$ and chooses two U(1) subgroups $\mathrm{U}(1)_1 \in G_1$ and $\mathrm{U}(1)_2 \in G_2$. As gauge group H one takes a product of the two U(1) groups, parametrized by ρ and τ, $H = \mathrm{U}(1)_\rho \times \mathrm{U}(1)_\tau$, with embeddings $\mathrm{em}_{\rho,1} : \mathrm{U}(1)_\rho \to \mathrm{U}(1)_1$, $\mathrm{em}_{\rho,2} : \mathrm{U}(1)_\rho \to \mathrm{U}(1)_2$, $\mathrm{em}_{\tau,1} : \mathrm{U}(1)_\tau \to \mathrm{U}(1)_1$, $\mathrm{em}_{\tau,2} : \mathrm{U}(1)_\tau \to \mathrm{U}(1)_2$. We assume that $\mathrm{U}(1)_1$ is generated by a_1, $\mathrm{U}(1)_1 = e^{i\lambda_1 a_1}$ and $\mathrm{U}(1)_2$ by a_2: $\mathrm{U}(1)_2 = e^{i\lambda_2 a_2}$ and the generators are normalized in the usual way, $\mathrm{Tr}a_1^2 = \mathrm{Tr}a_2^2 = 2$. The action of H we take in the form

$$(g_1, g_2) \to (h_1 g_1 h_2, h_2' g_2 h_1'))\,, \tag{3.1}$$

where

$$
\begin{aligned}
h_1 &= \mathrm{em}_{\rho,1}(h_\rho) = e^{ip\rho a_1}\,, \\
h_1' &= \mathrm{em}_{\rho,2}(h_\rho) = e^{iq\rho a_2}\,,
\end{aligned} \tag{3.2}
$$

$h_\rho \in \mathrm{U}(1)_\rho$, and

$$
\begin{aligned}
h_2 &= \mathrm{em}_{\tau,1}(h_\tau) = e^{ip\tau a_1}\,, \\
h_2' &= \mathrm{em}_{\tau,2}(h_\tau) = e^{iq\tau a_2}\,,
\end{aligned} \tag{3.3}
$$

where $h_\tau \in \mathrm{U}(1)_\tau$.

The action of the model in the absence of a boundary is

$$
S = S(g_1, k_1) + S(g_2, k_2) + S(g_1, g_2, A_1, A_2)\,, \tag{3.4}
$$

where $S(g_i, k_i)$, $i = 1, 2$ are the usual WZW actions given by (2.3) and $S(g_1, g_2, A_1, A_2)$ makes the action gauge invariant. Its explicit form is not important here for us and can be found in [11]. For gauge invariance, the levels k_1, k_2, and embedding coefficients p, q should satisfy

$$
k_1 p^2 = k_2 q^2\,. \tag{3.5}
$$

Now we consider the model in the presence of a boundary. We take the $\mathrm{U}(1)_\alpha$ group parametrized by α and consider embeddings $em_{\alpha,1} : \mathrm{U}(1)_\alpha \to \mathrm{U}(1)_1$, and $em_{\alpha,2} : \mathrm{U}(1)_\alpha \to \mathrm{U}(1)_2$. We define the boundary conditions

$$
g = (g_1, g_2)|_{\text{boundary}} = (m_1 C_1, m_2 C_2)\,, \tag{3.6}
$$

where

$$
\begin{aligned}
m_1 &= em_{\alpha,1}(m_\alpha) = e^{ip(\alpha+\gamma_1)a_1}\,, \\
m_2 &= em_{\alpha,2}(m_\alpha) = e^{iq(\alpha+\gamma_2)a_2}\,,
\end{aligned} \tag{3.7}
$$

and $m_\alpha \in \mathrm{U}(1)_\alpha$, $C_1 = l_1 f_1 l_1^{-1}$ and $C_2 = l_2 f_2 l_2^{-1}$. The parameters p and q are the same as in (3.2) and (3.3). γ_1 and γ_2 are possibly quantized [19] constants.

In other words, we take as the D-branes diagonally embedded U(1)s multiplied by the conjugacy classes. These boundary conditions were recently suggested in [10]. Our description (3.6) is slightly different from that in [10], and more convenient for present purposes.

Let us check that the boundary conditions (3.6) are invariant under the gauge transformation (3.1):

$$
\begin{aligned}
g &= (g_1, g_2) \to \left(h_1 g_1 h_2, h_2' g_2 h_1'\right) = \left(h_1 k l_1 f_1 l_1^{-1} h_2, h_2' k l_2 f_2 l_2^{-1} h_1'\right) \\
&= \Big((h_1 k h_2)\left(h_2^{-1} l_1\right) f_1 \left(h_2^{-1} l_1\right)^{-1}, (h_2' k h_1') \left(h_1^{'-1} l_2\right) f_2 \left(h_1^{'-1} l_2\right)^{-1} \Big)\,.
\end{aligned} \tag{3.8}
$$

We see that the boundary conditions preserve their form under the gauge transformation, with modified parameters:

$$
\alpha \to \alpha + \rho + \tau\,, \qquad l_1 \to h_2^{-1} l_1\,, \qquad l_2 \to h_1^{-1} l_2\,. \tag{3.9}
$$

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As explained in section 2, in the presence of a boundary the action should be modified by adding the boundary two-form [7]:

$$S = S(g_1, k_1) - \frac{k_1}{4\pi}\int_D \omega^{(2)}(m_1, l_1) + S(g_2, k_2) - \frac{k_2}{4\pi}\int_D \omega^{(2)}(m_2, l_2) + S(g_1, g_2, A_1, A_2)\,, \tag{3.10}$$

where $\omega^{(2)}(m, l)$ is defined by (2.9).

We now check that (3.10) is invariant under (3.1) accompanied by (3.9). First we compute the change of

$$S_1 = S(g_1, k_1) - \frac{k_1}{4\pi}\int_D \omega^{(2)}(m_1, l_1) \tag{3.11}$$

under the transformations $g_1 \to h_1 g_1 h_2$, $m_1 \to h_1 m_1 h_2$ and $l_1 \to h_2^{-1} l_1$, resulting from the presence of the boundary. From the Polyakov-Wiegmann identity we get

$$\begin{aligned}\Delta^{\rm bound} S(g_1, k_1) \;=\; -\frac{k_1}{4\pi}\int_D \mathrm{Tr}\Big(& h_1^{-1}dh_1 dm_1 m_1^{-1} + h_1^{-1}dh_1 dC_1 C_1^{-1} + h_1^{-1}dh_1 C_1 dh_2 h_2^{-1} C_1^{-1} + \\ & + C_1^{-1} m_1^{-1} dm_1 C_1 dh_2 h_2^{-1} + C_1^{-1} dC_1 dh_2 h_2^{-1}\Big)\,.\end{aligned} \tag{3.12}$$

Then we have

$$\Delta\omega^f(l) = \mathrm{Tr}\left(dh_2 h_2^{-1} C_1^{-1} dC_1 + dh_2 h_2^{-1} dC_1 C_1^{-1} + dh_2 h_2^{-1} C_1 dh_2 h_2^{-1} C_1^{-1}\right), \tag{3.13}$$

and

$$\begin{aligned}\Delta\left(\mathrm{Tr}\left(m_1^{-1} dm_1 dC_1 C_1^{-1}\right)\right) = \mathrm{Tr}\Big(& - h_1^{-1}dh_1 h_2^{-1} dh_2 + h_1^{-1} dh_1 dC_1 C_1^{-1} + \\ & + h_1^{-1} dh_1 C_1 dh_2 h_2^{-1} C_1^{-1} - \\ & - m_1^{-1} dm_1 h_2^{-1} dh_2 + m_1^{-1} dm_1 C_1 dh_2 h_2^{-1} C_1^{-1} + \\ & + h_2^{-1} dh_2 dC_1 C_1^{-1} + h_2^{-1} dh_2 C_1 dh_2 h_2^{-1} C_1^{-1}\Big).\end{aligned} \tag{3.14}$$

Collecting (3.12), (3.13) and (3.14) we obtain

$$\Delta^{\rm bound} S_1 = \frac{k_1}{4\pi}\int_D \mathrm{Tr}\left(h_2^{-1} dh_2 m_1^{-1} dm_1 - h_1^{-1} dh_1 dm_1 m_1^{-1} - h_1^{-1} dh_1 h_2^{-1} dh_2\right). \tag{3.15}$$

Similarly for S_2 we obtain

$$\Delta^{\rm bound} S_2 = \frac{k_2}{4\pi}\int_D \mathrm{Tr}\left(h_1'^{-1} dh_1' m_2^{-1} dm_2 - h_2'^{-1} dh_2' dm_2 m_2^{-1} - h_2'^{-1} dh_2' h_1'^{-1} dh_1'\right). \tag{3.16}$$

Taking into account (3.2), (3.3),(3.7) and (3.5) we find that $\Delta^{\rm bound} S_1 + \Delta^{\rm bound} S_2 = 0$, proving the gauge invariance of the action (3.10).

4. $\mathrm{SL}(2, \mathbb{R}) \times \mathrm{SU}(2)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ NW model

Let us consider the $\mathrm{SL}(2, \mathbb{R}) \times \mathrm{SU}(2)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ Nappi-Witten model.

Here $G_1 = \mathrm{SL}(2,\mathbb{R})$, $G_2 = \mathrm{SU}(2)$, $k_1 = -k_2$, $p = -i$, $q = 1$, and the $\mathrm{U}(1)_\rho \times \mathrm{U}(1)_\tau$ gauge group acts in the following way:

$$(g_1, g_2) \to (e^{\rho\sigma_3} g_1 e^{\tau\sigma_3}, e^{i\tau\sigma_3} g_2 e^{i\rho\sigma_3}) \,. \tag{4.1}$$

The D-branes proposed in section 3 have the form

$$g_{\text{boundary}} = (e^{(\alpha+\gamma_1)\sigma_3} C_1, e^{i(\alpha+\gamma_2)\sigma_3} C_2) \,, \tag{4.2}$$

where $C_1 = l_1 f_1 l_1^{-1}$ and $C_2 = l_2 f_2 l_2^{-1}$ are conjugacy classes, and f_2 belongs to the set (2.7). γ_1 and γ_2 are possibly quantized constants. Now we describe this hypersurface in detail. For this purpose we introduce Euler angles for $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SU}(2)$,

$$g_1 = e^{\chi_1 \frac{\sigma_3}{2}} e^{i\theta_1 \frac{\sigma_2}{2}} e^{\phi_1 \frac{\sigma_3}{2}} \,, \tag{4.3}$$
$$g_1 = e^{\chi_1 \frac{\sigma_3}{2}} e^{\tau_1 \frac{\sigma_1}{2}} e^{\phi_1 \frac{\sigma_3}{2}} \,, \tag{4.4}$$
$$g_2 = e^{i\chi_2 \frac{\sigma_3}{2}} e^{i\tilde{\theta}_2 \frac{\sigma_1}{2}} e^{i\phi_2 \frac{\sigma_3}{2}} \,, \tag{4.5}$$

where the first two formulae describe different patches of $\mathrm{SL}(2,\mathbb{R})$ and the last one is the usual Euler parametrisation for $\mathrm{SU}(2)$. It is shown in [7] that in the Euler angle parametrisations the product of a $\mathrm{U}(1)$ subgroup and a conjugacy class can be described by inequalities: $e^{\alpha\sigma_3} C_1$ in the patch given by (4.3) is described by the condition

$$\cos\frac{\theta_1}{2} \le \frac{\mathrm{Tr} f_1}{2} \,, \tag{4.6}$$

and in the patch (4.4) by the condition

$$\cosh\frac{\tau_1}{2} \le \frac{\mathrm{Tr} f_1}{2} \,, \tag{4.7}$$

and $e^{i\alpha\sigma_3} C_2$ in the parametrisation (4.5) is given by the condition

$$\cos\frac{\tilde{\theta}_1}{2} \ge \frac{\mathrm{Tr} f_2}{2} \,. \tag{4.8}$$

In order to find the equation of the D-brane hypersurface we should find α on the $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SU}(2)$ sides and equate them to each other. It is easy to find the angle α in each case. Writing the boundary condition in the form $e^{-\alpha\sigma_3} g_1 = C_1$ and taking the trace on both sides we easily obtain in the first patch:

$$\cosh\left(\alpha + \gamma_1 - \frac{\chi_1 + \phi_1}{2}\right) = \frac{\mathrm{Tr} f_1}{2\cos\frac{\theta_1}{2}} \,, \tag{4.9}$$

in the second patch:

$$\cosh\left(\alpha + \gamma_1 - \frac{\chi_1 + \phi_1}{2}\right) = \frac{\mathrm{Tr} f_1}{2\cosh\frac{\tau_1}{2}} \,, \tag{4.10}$$

and for $\mathrm{SU}(2)$:

$$\cos\left(\alpha + \gamma_2 - \frac{\chi_2 + \phi_2}{2}\right) = \frac{\mathrm{Tr} f_2}{2\cos\frac{\tilde{\theta}_2}{2}} \,. \tag{4.11}$$

We see that the conditions (4.6), (4.7) and (4.8) are necessary for the existence of solutions to eq. (4.9),(4.10) and (4.11) respectively. Now using gauge fixing conditions $\chi_1 = 0$ and $\phi_1 = 0$ we can explicitly write down the D-brane hypersurface equation. In the first patch we have

$$\cosh\left(\arccos\left(\frac{\mathrm{Tr} f_2}{2\cos\frac{\tilde{\theta}_2}{2}}\right) + \frac{\chi_2+\phi_2}{2} + \gamma_2 - \gamma_1\right) = \frac{\mathrm{Tr} f_1}{2\cos\frac{\theta_1}{2}}\,, \tag{4.12}$$

and in the second patch

$$\cosh\left(\arccos\left(\frac{\mathrm{Tr} f_2}{2\cos\frac{\tilde{\theta}_2}{2}}\right) + \frac{\chi_2+\phi_2}{2} + \gamma_2 - \gamma_1\right) = \frac{\mathrm{Tr} f_1}{2\cosh\frac{\tau_1}{2}}\,. \tag{4.13}$$

5. D-branes in the Guadagnini-Martellini-Mintchev model

We begin by reviewing the model introduced in [12] and [13] (see also [15]). This model is a kind of gauged WZW model based on a group $G_1 \times G_2$. The gauge group H acts in the folowing way: we choose subgroups $H_1 \in G_1$ and $H_2 \in G_2$ and take embeddings $em_1 \ : \ H \to H_1$ and $em_2 \ : \ H \to H_2$. It is assumed that H_1 and H_2 are the same subgroups of G_1 and G_2 :$H = H_1 = H_2$. The group H acts by the formula

$$(g_1, g_2) \to \left(g_1 em_1(h^{-1}), em_2(h) g_2\right)\,. \tag{5.1}$$

It was shown in [12] that the following action is invariant under (5.1):

$$S_{\mathrm{GMM}} = S(g_1, k_1) + S(g_2, k_2) + S_{\mathrm{int}}(g_1, g_2, k)\,, \tag{5.2}$$

where $S(g_i, k_i)$, $i = 1, 2$ are the usual WZW actions (2.3) and

$$\begin{aligned} S_{\mathrm{int}}(g_1, g_2, k) \;=\; -\frac{k}{2\pi}\int d^2x \Big(&\mathrm{Tr}\left(R_\alpha g_1^{-1}\partial_\mu g_1\right)\mathrm{Tr}\left(R'_\alpha \partial^\mu g_2 g_2^{-1}\right) + \\ &+ \epsilon^{\mu\nu}\mathrm{Tr}\left(R_\alpha g_1^{-1}\partial_\mu g_1\right)\mathrm{Tr}\left(R'_\alpha \partial_\nu g_2 g_2^{-1}\right)\Big). \end{aligned} \tag{5.3}$$

Here R_α and R'_α are the generators of the Lie algebra of the subgroup H in G_1 and G_2 respectively. It is shown in [12] that for gauge invariance the coefficients entering in (5.2) should satisfy

$$k_1 = kr'\,, \qquad k_2 = kr\,, \tag{5.4}$$

where r and r' are given by the embeddings:

$$\mathrm{Tr}(R_\alpha R_\beta) = r\delta_{\alpha\beta}\,, \qquad \mathrm{Tr}(R'_\alpha R'_\beta) = r'\delta_{\alpha\beta}\,. \tag{5.5}$$

The conformal field theory defined by this sigma model was discussed in [13], where the current algebra and the Virasoro algebra with a central charge value coinciding with that of the GKO construction for the coset $(G_1 \times G_2)/H$ were found.

Here we consider the case when the gauge group is an abelian group, parametrized by ρ: $H = \mathrm{U}(1)_\rho$. As before we assume that H_1 is generated by a generator a_1, $H_1 = e^{i\lambda_1 a_1}$ and

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H_2 by a_2: $H_2 = e^{i\lambda_2 a_2}$, and that the generators are normalized as usual: $\mathrm{Tr} a_1^2 = \mathrm{Tr} a_2^2 = 2$. In this case the gauge group acts as

$$(g_1, g_2) \to (g_1 h_1, h_2 g_2) , \tag{5.6}$$

where

$$\begin{aligned} h_1 &= em_1(h_\rho^{-1}) = e^{-ip\rho a_1} , \\ h_2 &= em_2(h_\rho) = e^{iq\rho a_2} , \end{aligned} \tag{5.7}$$

$h_\rho \in \mathrm{U}(1)_\rho$ and p and q satisfy the relation

$$k_1 p^2 = k_2 q^2 . \tag{5.8}$$

Now we consider the model in the presence of a boundary. We take the $\mathrm{U}(1)_\alpha$ group parametrized by α and consider embeddings $em_{\alpha,1} : \mathrm{U}(1)_\alpha \to \mathrm{U}(1)_1$, and $em_{\alpha,2} : \mathrm{U}(1)_\alpha \to \mathrm{U}(1)_2$. We suggest the following boundary conditions:

$$(g_1, g_2)|_{\text{boundary}} = (m_1 C_1, m_2 C_2) , \tag{5.9}$$

where

$$\begin{aligned} m_1 &= em_{\alpha,1}(m_\alpha) = e^{-ip(\alpha+\gamma_1)a_1} , \\ m_2 &= em_{\alpha,2}(m_\alpha) = e^{iq(\alpha+\gamma_2)a_2} \end{aligned} \tag{5.10}$$

and $m_\alpha \in \mathrm{U}(1)_\alpha$, $C_1 = l_1 f_1 l_1^{-1}$, $C_2 = l_2 f_2 l_2^{-1}$. The parameters p and q are the same as in (5.7). γ_1 and γ_2 are possibly quantized [19] constants. These boundary conditions are invariant under (5.1):

$$\begin{aligned} &\left(m_1 l_1 f_1 l_1^{-1}, m_2 l_2 f_2 l_2^{-1}\right) \to \left(m_1 l_1 f_1 l_1^{-1} h_1, h_2 m_2 l_2 f_2 l_2^{-1}\right) = \\ &= \left((h_1 m_1)(h_1^{-1} l_1) f_1 (h_1^{-1} l_1)^{-1}, (h_2 m_2) l_2 f_2 l_2^{-1}\right) . \end{aligned} \tag{5.11}$$

We see that boundary conditions keep the form with modified parameters

$$\alpha \to \alpha + \rho , \qquad l_1 \to h_1^{-1} l_1 . \tag{5.12}$$

In the presence of a boundary we suggest the following action:

$$S_{\mathrm{GMM}} = S(g_1, k_1) - \frac{k_1}{4\pi} \int_D \omega^{(2)}(m_1, l_1) + S(g_2, k_2) - \frac{k_2}{4\pi} \int_D \omega^{(2)}(m_2, l_2) + S_{\mathrm{int}}(g_1, g_2, k) . \tag{5.13}$$

Now we check that the action is invariant under (5.1) accompanied by (5.12). From the formula (3.16) we easily derive the change of S_1 and S_2 under a gauge transformation,

$$\Delta^{\text{boundary}} S_1 = \frac{k_1}{4\pi} \int_D \mathrm{Tr}\left(h_1^{-1} dh_1 dm_1 m_1^{-1}\right) , \tag{5.14}$$

$$\Delta^{\text{boundary}} S_2 = -\frac{k_2}{4\pi} \int_D \mathrm{Tr}\left(h_2^{-1} dh_2 dm_2 m_2^{-1}\right) . \tag{5.15}$$

which cancel each other as a consequence of the conditions (5.7), (5.10) and (5.8).

6. $\mathrm{SU}(2)\times\mathrm{SU}(2)/\mathrm{U}(1)$ GMM model

We begin by describing this model following [15].

The SU(2) group elements are parametrized as

$$\begin{aligned} g_1 &= \exp(i\phi_1\sigma_3)\exp(i\theta_1\sigma_2)\exp(i\psi_1\sigma_3)\,, \\ g_2 &= \exp(i\phi_2\sigma_3)\exp(i\theta_2\sigma_2)\exp(i\psi_2\sigma_3)\,. \end{aligned} \tag{6.1}$$

The gauge action of the U(1) subgroup is defined by

$$\psi_1 \to \psi_1 - p\varepsilon(z,\bar z)\,, \qquad \phi_2 \to \phi_2 + q\varepsilon(z,\bar z)\,. \tag{6.2}$$

In the parametrization (6.1) the action (5.2) is

$$\begin{aligned} S = \frac{1}{4\pi}\int d^2x \Big[& k_1(\partial_\mu\theta_1\partial^\mu\theta_1 + \partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\psi_1\partial^\mu\psi_1 + \cos(2\theta_1)\partial_\mu\phi_1\partial_\nu\psi_1(\eta^{\mu\nu}+\epsilon^{\mu\nu})) + \\ & + k_2\,(\partial_\mu\theta_2\partial^\mu\theta_2 + \partial_\mu\phi_2\partial^\mu\phi_2 + \partial_\mu\psi_2\partial^\mu\psi_2 + \cos(2\theta_2)\partial_\mu\phi_2\partial_\nu\psi_2(\eta^{\mu\nu}+\epsilon^{\mu\nu})) + \\ & + k_3(\cos(2\theta_1)\partial_\mu\phi_1 + \partial_\mu\psi_1)(\cos(2\theta_2)\partial_\nu\psi_2 + \partial_\mu\phi_2)(\eta^{\mu\nu}+\epsilon^{\mu\nu})\Big]\,. \end{aligned} \tag{6.3}$$

For the action to be invariant under (6.2) one needs to impose the following algebraic constraints:

$$k_1 p = k_3 q\,, \qquad k_2 q = k_3 p\,. \tag{6.4}$$

Multiplying these equation we obtain

$$k_3 = \sqrt{k_1 k_2}\,, \qquad \frac{p}{q} = \sqrt{\frac{k_2}{k_1}}\,. \tag{6.5}$$

Fixing the gauge by setting $\phi_2 = 0$ one gets a background whose metric is of the (non-Einstein) $T^{1,Q}$ type

$$ds^2 = k\left[d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 + Q^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) + (d\psi + \cos\theta_1 d\phi_1 + Q\cos\theta_2 d\phi_2)^2\right]\,, \tag{6.6}$$

where we have rescaled all variables by 1/2, renamed $\psi_2 \to \phi_2$, $\psi_1 \to \psi$ and introduced

$$Q = \frac{p}{q} = \sqrt{\frac{k_2}{k_1}}\,, \qquad k = k_1\,. \tag{6.7}$$

The background also includes the antisymmetric tensor field

$$B_{\phi_1\psi} = k\cos\theta_1\,, \qquad B_{\phi_1\phi_2} = kQ\cos\theta_1\cos\theta_2\,, \qquad B_{\phi_2\psi} = -kQ\cos\theta_2\,. \tag{6.8}$$

Now we are ready to present D-branes in this background and to compute the DBI action.

The D-branes proposed in section 5 have the form

$$(g_1, g_2)_{\text{boundary}} = \left(e^{-ip(\alpha+\gamma_1)\sigma_3}C_1, e^{iq(\alpha+\gamma_2)\sigma_3}C_2\right)\,, \tag{6.9}$$

where $C_1 = h_1 f_1 h_1^{-1}$ and $C_2 = h_2 f_2 h_2^{-1}$ are conjugacy classes, $f_1 = e^{i\hat\psi_1\sigma_3}$ and $f_2 = e^{i\hat\psi_2\sigma_3}$, and $\hat\psi_1$, $\hat\psi_2$ belong to the set (2.8). Let us now find the equation describing this hypersurface. As before, we should find in the parametrization (6.1) the angle α and equate both sides. Writing the boundary conditions as

$$\mathrm{Tr}(e^{ip(\alpha+\gamma_1)\sigma_3}g_1) = 2\cos\hat{\psi}_1\,, \tag{6.10}$$

$$\mathrm{Tr}(e^{-iq(\alpha+\gamma_2)\sigma_3}g_2) = 2\cos\hat{\psi}_2\,, \tag{6.11}$$

from (6.10) and (6.11) we obtain

$$\cos(p(\alpha+\gamma_1)+\phi_1+\psi_1) = \frac{\cos\hat{\psi}_1}{\cos\theta_1}\,, \tag{6.12}$$

and

$$\cos(-q(\alpha+\gamma_2)+\phi_2+\psi_2) = \frac{\cos\hat{\psi}_2}{\cos\theta_2}\,. \tag{6.13}$$

Eliminating α from (6.12) and (6.13) we get

$$\frac{1}{p}\arccos\left(\frac{\cos\hat{\psi}_1}{\cos\theta_1}\right) - \frac{\phi_1+\psi_1}{p} - \gamma_1 = -\frac{1}{q}\arccos\left(\frac{\cos\hat{\psi}_2}{\cos\theta_2}\right) + \frac{\phi_2+\psi_2}{q} - \gamma_2\,. \tag{6.14}$$

Using now the gauge fixing condition $\phi_2 = 0$, and rescaling and renaminig all the variables as before, we get the D-brane hypersurface on this $T^{1,Q}$ type space,

$$\phi_2 = 2\arccos\left(\frac{\cos\hat{\psi}_2}{\cos\frac{\theta_2}{2}}\right) + \frac{2}{Q}\arccos\left(\frac{\cos\hat{\psi}_1}{\cos\frac{\theta_1}{2}}\right) - \frac{\phi_1+\psi}{Q} + 2q(\gamma_2-\gamma_1)\,, \tag{6.15}$$

where Q is defined in (6.7). As before θ_1 and θ_2 satisfy the inequalities

$$\cos\frac{\theta_1}{2} \geq \cos\hat{\psi}_1\,, \qquad \cos\frac{\theta_2}{2} \geq \cos\hat{\psi}_2\,. \tag{6.16}$$

The presence of the constant term $q(\gamma_2-\gamma_1)$ reflects the invariance of the action (6.3) under the rotations $\phi_i \to \phi_i + \beta_i$, $\psi_i \to \psi_i + \delta_i$, where β_i and δ_i are constant angles, $i = 1,2$. But, as noted in [19], in the gauged WZW models these symmetries are broken to some discrete subgroups. In the case in question we have

$$\gamma_1 = \frac{n_1}{k_1 p^2}\,, \qquad \gamma_2 = \frac{n_2}{k_2 q^2}\,, \tag{6.17}$$

where n_1 and n_2 are integers, and using (6.5) we have for the last part

$$2q(\gamma_1-\gamma_2) = \frac{2n}{qk_2}\,, \tag{6.18}$$

where $n = n_1 - n_2$. We see that the branes (6.15) are specified by the three parameters $\hat{\psi}_1$, $\hat{\psi}_2$ and n, in one-to-one correspondence with the primaries of the corresponding GKO coset model $(\mathrm{SU}(2)\times\mathrm{SU}(2))/\,\mathrm{U}(1)$. The last piece that we need for the computation of the mass of the D-branes is the field-strength. It can be derived from the field-strength for the corresponding branes on groups by imposing the gauge fixing condition. Using the formula (2.11) we have:

$$F = \frac{k\tan\frac{\theta_1}{2}\cos\hat{\psi}_1}{\sqrt{\cos^2\frac{\theta_1}{2}-\cos^2\hat{\psi}_1}}d\theta_1\wedge(d\phi_1-d\psi) - \frac{kQ^2\tan\frac{\theta_2}{2}\cos\hat{\psi}_2}{\sqrt{\cos^2\frac{\theta_2}{2}-\cos^2\hat{\psi}_2}}d\theta_2\wedge d\phi_2 - kd\phi_1\wedge d\psi\,. \tag{6.19}$$

For further use we note that using (6.15) we can compactly re-write (6.19) as

$$F = -kQ\partial_{\theta_1}\phi_2 d\theta_1 \wedge (d\phi_1 - d\psi) + kQ^2\partial_{\theta_2}\phi_2 d\theta_2 \wedge d\phi_2 - kd\phi_1 \wedge d\psi\,. \tag{6.20}$$

We turn to the computation of the DBI action

$$S = \int \sqrt{\det \phi_2^*(g+B+F)}\,, \tag{6.21}$$

where ϕ_2 is the embedding (6.15) of the D-brane into the target space. We first compute the induced metric on the D-brane. Inserting (6.15) in (6.6) we obtain the following elements of the induced metric $G = \phi_2^* g$:

$$\begin{aligned} G_{\theta_1\theta_1} &= k + kQ^2(\partial_{\theta_1}\phi_2)^2 \quad G_{\theta_1\theta_2} = kQ^2\partial_{\theta_1}\phi_2\partial_{\theta_2}\phi_2 \quad G_{\theta_1\phi_1} = kQ(\cos\theta_1\cos\theta_2 - 1)\partial_{\theta_1}\phi_2 \\ G_{\theta_1\psi} &= kQ\partial_{\theta_1}\phi_2(\cos\theta_2 - 1) \quad G_{\theta_2\theta_2} = kQ^2(1+(\partial_{\theta_2}\phi_2)^2) \quad G_{\theta_2\psi} = kQ\partial_{\theta_2}\phi_2(\cos\theta_2 - 1) \\ G_{\theta_2\phi_1} &= kQ(\cos\theta_1\cos\theta_2 - 1)\partial_{\theta_2}\phi_2 \quad G_{\phi_1\phi_1} = 2k(1-\cos\theta_1\cos\theta_2) \\ G_{\phi_1\psi} &= k + k\cos\theta_1 - k\cos\theta_2 - k\cos\theta_1\cos\theta_2 \quad G_{\psi\psi} = 2k(1-\cos\theta_2)\,. \end{aligned} \tag{6.22}$$

Inserting (6.15) in (6.8) and (6.19) and adding this to (6.22) we obtain the following matrix $\phi_2^*(g+B+F)$:

$$\begin{pmatrix} k+kQ^2(\partial_{\theta_1}\phi_2)^2 & 0 & -2kQ\partial_{\theta_1}\phi_2 & 0 \\ 2kQ^2\partial_{\theta_1}\phi_2\partial_{\theta_2}\phi_2 & kQ^2(1+(\partial_{\theta_2}\phi_2)^2) & -2kQ\partial_{\theta_2}\phi_2 & -2kQ\partial_{\theta_2}\phi_2 \\ 2kQ\partial_{\theta_1}\phi_2\cos\theta_1\cos\theta_2 & 2kQ\partial_{\theta_2}\phi_2\cos\theta_1\cos\theta_2 & 2k(1-\cos\theta_1\cos\theta_2) & 2k\cos\theta_1(1-\cos\theta_2) \\ 2kQ\partial_{\theta_1}\phi_2(\cos\theta_2-1) & 2kQ\partial_{\theta_2}\phi_2\cos\theta_2 & 2k(1-\cos\theta_2) & 2k(1-\cos\theta_2) \end{pmatrix}. \tag{6.23}$$

Computing the determinant we obtain

$$\det\phi_2^*(g+B+F) = 16k^4Q^2\left(\sin^2\frac{\theta_1}{2} + Q^2(\partial_{\theta_1}\phi_2)^2\cos^2\frac{\theta_1}{2}\right)\left(\sin^2\frac{\theta_2}{2} + (\partial_{\theta_2}\phi_2)^2\cos^2\frac{\theta_2}{2}\right). \tag{6.24}$$

Finally, expanding the derivatives we get

$$\sqrt{\det\phi_2^*(g+B+F)} = 4k^2Q\frac{\sin\theta_1\sin\theta_2}{\sqrt{2(\cos\theta_1 - \cos 2\hat{\psi}_1)}\sqrt{2(\cos\theta_2 - \cos 2\hat{\psi}_2)}}\,. \tag{6.25}$$

Now integrating (6.25) we obtain the energy of the brane:

$$E_{\mathrm{DBI}} = \int_0^{2\hat{\psi}_1} d\theta_1 \int_0^{2\hat{\psi}_1} d\theta_2 \int_0^{4\pi} d\phi_1 \int_0^{\pi} d\psi \sqrt{\det\phi_2^*(g+B+F)} = 64k^2Q\pi^2\sin\hat{\psi}_1\sin\hat{\psi}_2\,. \tag{6.26}$$

Let us compare this result with the CFT predictions. In a rational CFT the Cardy boundary states describing the D-branes are

$$|a\rangle_C = \sum \frac{S_{ab}}{\sqrt{S_{0b}}}|b\rangle\rangle\,, \tag{6.27}$$

where a and b label primaries, $|b\rangle\rangle$ are the Ishibashi states, and S_{ab} is the modular-transformation matrix. The mass of a Cardy state is given by the coefficient of its $b = 0$ Ishibashi component [20, 21]:

$$M_C \sim \frac{S_{a0}}{\sqrt{S_{00}}} \,. \tag{6.28}$$

In the coset $(\mathrm{SU}(2) \times \mathrm{SU}(2))/\,\mathrm{U}(1)$ the modular-transformation matrix is the product of the corresponding matrices of all the constituent groups. The S-matrix of SU(2) at level k is

$$S_{ij} = \sqrt{\frac{2}{k+2}} \sin\left(\frac{(2i+1)(2j+1)\pi}{k+2}\right) . \tag{6.29}$$

We want to compare the CFT mass prediction with the DBI result in the semiclassical limit of large k. In this limit

$$\frac{S_{i0}}{\sqrt{S_{00}}} = \frac{(2k+4)^{1/4}}{\pi} \sin\hat{\psi} \,, \tag{6.30}$$

where $\hat{\psi}$ is defined in (2.8). Collecting everything we obtain that according to the CFT computations in the semiclassical limit the mass of the D-brane specified by $\hat{\psi}_1$ and $\hat{\psi}_2$ is

$$M_C \sim \sin\hat{\psi}_1 \sin\hat{\psi}_2 \tag{6.31}$$

in agreement with the DBI result (6.26).

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